DH-003-001616 Seat No. _______

B. Sc. (Sem. VI) (CBCS) Examination

March-2022

Mathematic : BSMT-601(A)

(Graph Theory & Complex Analysis-2)

(Old Course)

Faculty Code : 003
Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Figures to the right indicates marks.

1 Answer the following questions : **20**

(1) Define degree of a vertex in a graph.

(2) Give an example of a complete bipartite graph which is isomorphic to cycle graph with 4 vertices.

(3) Define unilaterally connected graph.

(4) Complete graph K_n os Euler graph if and ony if n is even. (True/False)

(5) Number of edge-disjoint Hamiltonian circuits in complete graph K_9 is _______.

(6) What is chromatic mumber of a bipartite graph ?

(7) Define binary tree.

(8) What is the number of fundamental cut-sets in any connected graph ?

(9) Vertex connectivity of complete bipartite graph $K_{m,n}$ is________.

(10) Number of faces in a connected planar graph with n vertices and e edges is ________.

(11) A series $\sum_{n=0}^{\infty}\frac{1}{1-z}$ is convergent for _______.

(12) Every convergent series is aboslutely convergent. (True/False)

(13) Write Maclaurian series of e^z.

(14) Define zeros of a complex function.

(15) Define essential singular point.

(16) What is the residue of $\frac{\sin z}{z}$ $at\ z_0 = 0$?

(17) The value of $\int_C e^{\frac{1}{z}} dz$ is________.

(18) Define conformal mapping.

(19) Write a linear transformation that reflect expansion or contraction.

(20) What is genral form of a bilinear transformation ?

2 (A) Attempt any **three** : **6**

(1) Prove that in any graph G the number of odd vertices is always even.

(2) What will be the resultant graph by using the following operations ?

(a) $K_n \setminus \{\upsilon\}$, where υ is any vertex of K_n.

(b) $G \oplus G$, where G is any graph.

(3) Find the number of pendant vertices in any binary tree on n vertices.

(4) Drwa (geometric) dual of the graph K_4.

(5) Find rank and nullity of $K_{2,3}$.

(6) (a) What is the dimension of circuit subspace ?

(b) What is the dimension of cut-set subspace ?

2 (B) Attempt any **three** : **9**

(1) Prove that a graph G is a tree if and only if G is minimally connected.

(2) Prove that mumber of edge-disjoint Hamiltonian circuits in K_n is $\frac{n-1}{2}$, where n is odd and $n \geq 3$.

(3) Derive formula to obtain minimum height for any binary tree on n vertices.

(4) For any connected planar bipartite graph with n vertices and e edges, prove that $e \leq 2n - 4$

(5) Prove that vertex connectivity of any connected graph G with n vertices and e edges is less than or equal to $\left[\frac{2e}{n}\right]$

(6) Prove that the rank of incidence matrix of any graph on n vertices is $n-1$.

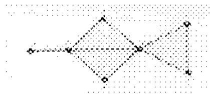
(C) Attempt any **Two** : **10**

(1) Prove that a graph G is disconnected if and only if vertex set of G can be partitioned into two nonempty, disjoint subsets V_1 and V_2 such that there is no edge in G with one end vertex in V_1 and other end vertex in V_2.

(2) Prove that the maximum number of edges in any simple graph with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$

(3) Prove that a connected graph G is an Euler graph if and only if each vertex of G is even.

(4) Find all maximal independent sets and hence determine the chromatic partitions of the following graph.



(5) Prove that $(W_G, +_2, \cdot_2)$ is a vector space over $\mathbb{Z}_2$.

3 (A) Attempt any **Three** : **6**

(1) If $Z_n = \frac{1}{n^3} + i$, then using definition show that $\lim_{n\to\infty} Z_n = i$.

(2) Find the Maclaurian's series of $z\cos(z)$.

(3) Prove that $f(z) = e^z$ is conformal mapping at each point of Z-plane.

(4) Describe transformation $w = z + a$.

(5) Evaluate $\int_C \frac{1}{z(z-2)} dz, C : |z| = 3.$.

(6) Find residue of $\cot z \ zt \ z = 0$.

(C) Attempt any **Three** : **9**

(1) For $|z| < 1$, prove that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$.

(2) Expand sin z into a Taylor's series about the point $z_0 \frac{\pi}{2}$.

(3) Find Laurent's series expansion in powers of z for the function $f(z) \frac{1}{(z^2+1)(z+2)}$ in the region $1 < |z| < 2$.

(4) Discuss the types of isolated singilarities with suitable examples.

(5) Determine fixed points and critical points of the bilinear transformation $\omega = \frac{z-1}{z+1}$.

(6) Show that the linear transformation $\omega = 1/z$ maps the circle $|z - 2i| = 2$ into the straight line $4\upsilon + 1 = 0$.

(C) Attempt any **Two** : **10**

(1) State and prove Taylor's theorem.

(2) State and prove Cauchy's residue theorem.

(3) Evaluate $\int_C \frac{3z^3 + 2}{(z-1)(z^2+9)} dz; C: |z| = 4$.

(4) Show that $\int_0^\infty \frac{x \sin ax}{x^2 + k^2} dx = \frac{\pi}{2} e^{-ak}$ $(a > 0, k > 0)$

(5) Find linear fractional transformation which maps the points $z_1 = -1, z_2 = 0, z_3 = 1$ onto the points $w_1 = -i, w_2 = 1, w_3 = i$.